Reg. No. :

Question Paper Code : X62724

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020

First Semester

Civil Engineering

MA1101 – MATHEMATICS – I

(Common to all Branches)

(Regulations 2004/2007/2008)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. Find the sum of the Eigen values of the inverse of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$.

2. Find the characteristic equation of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

3. Find the equation of the sphere having the points (0, 0, 0) and (2, –2, 4) as ends of the diameter.

4. If $\frac{x}{3} = \frac{y}{4} = \frac{z}{k}$ is a generator of the cone $x^2 + y^2 - z^2 = 0$, then find the value of k.

5. Find the radius of curvature of the curve $x = t^2$, $y = t$ at $t = 1$.

6. Find the equation of the envelope for the family of straight line $y = mx + am^2$.

7. Find the stationary point of $f(x, y) = x^2 - xy + y^2 - 2x + y$.

8. Find $\frac{du}{dt}$, if $u = xy + yz + zx$, where $x = e^t$, $y = e^{-t}$ and $z = \frac{1}{t}$.

9. Solve $y''' - y'' - y' + y = 0$.

10. Convert the equation $x^2y'' - xy' + y = 2\log x$ as a linear equation with constant co-efficients.

PART – B **(5×16=80 Marks)**

11. a) i) Find all the Eigen values and Eigen Vectors of the matrix $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$. **(8)**

ii) Diagonalise the matrix $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ by similarity transformation. **(8)**

(OR)

b) i) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ by using Cayley Hamilton theorem. **(8)**

ii) Obtain an orthogonal transformation, which will transform the quadratic form $2x_1^2 + 6x_2^2 + 2x_3^2 + 8x_1x_3$ into a canonical form. **(8)**

12. a) i) Find the equation to the right circular cylinder with radius 5 and whose axis is $\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$. **(8)**

ii) Find center and radius of the circle given by $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ and $x + 2y + 2z + 7 = 0$. **(8)**

(OR)

b) i) Find the equation to the sphere having the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$; $2x + 3y + 4z = 8$ as a great circle. **(8)**

ii) Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = y - 4 = \frac{z-5}{3}$ are coplanar. Find the plane containing them. **(8)**

13. a) i) Find the radius of the curvature for the curve $y^2 = 12x$ at (3, 6). **(8)**

ii) Find the equation of the envelope for the family of the lines $\frac{x}{a} + \frac{y}{b} = 1$ with the condition on the parameters a + b = c for a constant C. **(8)**

(OR)

b) i) Find the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at a point (a, a). **(8)**

ii) Find the evolute of the parabola $y^2 = 4ax$, considering it as the envelope of its normals. **(8)**

14. a) i) Examine the function $f(x, y) = x^4 - y^4 - 2x^2 + 2y^2 + 10$ for maximum and minimum values. **(8)**

ii) Find Taylor's series expansion of $f(x, y) = x^y$ about (1, 1) up to third degree terms. **(8)**

(OR)

b) i) Let $\phi(x, y)$ be a two variable function which is twice differentiable with respect to x and y variables. Under the co-ordinate transformation $u = x^2 - y^2$ and $v = 2xy$ show that

$$\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) = 4\left(x^2 + y^2\right)\left(\frac{\partial^2\phi}{\partial u^2} + \frac{\partial^2\phi}{\partial v^2}\right).$$ **(8)**

ii) Suppose a closed rectangular box has length twice its breadth and has constant volume of 72 cubic units. Using Lagrange multiplier technique, determine the dimension of the box requiring least metal sheets for its construction. **(8)**

15. a) i) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$. **(8)**

ii) Solve the following simultaneous equations.
$(2D - 3)x + Dy = e^t$, $Dx + (D + 2)y = \cos 2t$. **(8)**

(OR)

b) i) Solve $(x^2 D^2 + xD + 1)y = 4 \sin(\log x)$. **(8)**

ii) Solve by the variation of parameters method $\frac{d^2y}{dx^2} + 4y = \tan 2x$. **(8)**
